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FREE TRANSVERSE VIBRATIONS OF AN ELASTICALLY CONNECTED RECTANGULAR SIMPLY SUPPORTED DOUBLE-PLATE COMPLEX SYSTEM

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In this paper, the free transverse vibrations of a system of two rectangular simply supported thin plates connected by a homogeneous Winkler elastic layer are investigated analytically. The small vibrations of the system are described by a set of two partial differential equations, based on the Kirchhoff–Love plate theory. Next, the homogeneous equations of motion are solved by using the classical Navier method. The natural frequencies of the system in the form of two infinite sequences are determined and the corresponding mode shapes of vibration are shown. As a consequence, an elastically connected double-plate complex system executes two kinds of the free vibrations: synchronous and asynchronous. The initial-value problem is then considered to find the final form of the free vibrations. The theoretical analysis presented is illustrated by a numerical example, in which the free vibrations of a system of two identical plates are discussed in detail.

1. INTRODUCTION

Plate-type structures are of great importance in many fields of civil and mechanical engineering. The present paper deals with a complex system of two rectangular plates which are connected by a homogeneous Winkler elastic layer. This complex continuous system can be used as a model for a three-layered structure consisting of three plates, in which the middle light layer is represented by means of a Winkler-type foundation. The vibration analysis of such a system is possible and not mathematically complicated for certain particular cases of the boundary conditions; therefore it can be carried out by using the same procedures as those used for a single plate. In the plate vibration theory based on the classical Kirchhoff–Love assumptions, two basic analytical methods are applied for determining the free vibrations of a single thin rectangular plate [1–7]. These are the Lévy and Navier methods. As is well known, the Lévy method [1–9, 11, 13–19] is used for a plate having two opposite parallel edges simply supported and arbitrary boundary conditions on the other two edges. The Navier method [1–7, 10, 12, 15, 16, 18, 19], being a particular case of the Lévy method, is appropriated for analyzing a plate with all four edges simply supported. In this work, the Navier procedure is applied for formulating the complete exact theoretical solutions for the free transverse vibrations of a simply supported rectangular double-plate system.

The vibration problems of an elastically connected double-plate complex system have been a subject of a number of papers. The first significant work, by Kunukkasseril and Radhakrishnan [20], was devoted to the free vibrations of a rectangular multi-plate system.

The simple case of two identical plates was worked out in detail. The explicit forms of the frequency equations and mode shapes of vibration were determined for five combinations of the basic Lévy-type boundary conditions. The general free and forced vibration theory formulated for two different rectangular plates elastically joined has been developed by the author in his early papers [21–23]. Applying the Green's function method, Kukla [24–29] has considered the free vibrations of two elastically connected plates assuming three different models of an elastic layer as: (1) a system of discrete translational elastic elements [24, 29], (2) a system of line distributed translational elastic elements [25, 27, 29], and (3) a non-homogeneous continuous elastic layer [26, 28, 29]. The forced vibrations of a rectangular double-plate system subjected to a moving line load has been investigated by Chonan [30, 31]. A similar problem of the response of two plates under a moving concentrated force has been examined by Szcześniak [32, 33]. References [7, 34, 35], devoted to applying a double-plate system as a continuous dynamic vibration absorber (CDVA), are especially interesting because of the great practical importance of these devices. Certain dynamical problems of the rectangular plate systems have been also treated in, for example, references [36–41].

The transverse vibrations of elastically connected circular multi-plate and double-plate complex systems have been studied by many authors, including, among others, Kunukkasseril and Swamidas [42–46], Kunukkasseril and Venkatesan [47, 48], Chonan [49, 50], Moghilevskiy [51], and Oniszczuk [52, 53].

It is relevant to note that the vibration analysis of double-membrane systems presented by the author in his papers [54–61] can be helpful in the investigations of analogous systems of simply supported plates.

In this publication the free vibration analysis is limited only to the system of two plates having all edges simply supported. The vibrations of a more general plate system with the boundary conditions of the Lévy type will be discussed by the author in later papers.

2. FORMULATION OF THE PROBLEM

The physical model of the vibrating system is composed of two parallel rectangular plates connected by a Winkler elastic layer (see Figure 1). It is assumed that the plates are thin, homogeneous and isotropic, and that they have constant thickness. Only the case of the plates having all edges simply supported is considered. In general, it is also assumed that the plates are subjected to transverse arbitrarily distributed continuous loads. The small undamped vibrations of the system are analyzed.

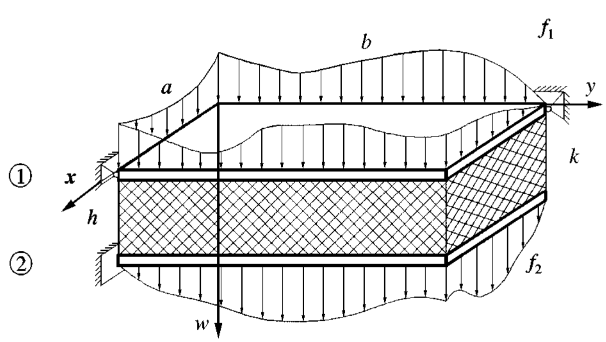

Figure 1. The general model of an elastically connected rectangular double-plate complex system with arbitrary boundary conditions.

The transverse vibrations of an elastically connected rectangular double-plate system are governed by the following differential equations [7, 14], based on the classical Kirchhoff–Love plate theory:

$$D_1 \Delta^2 w_1 + m_1 \ddot{w}_1 + k(w_1 - w_2) = f_1, \quad D_2 \Delta^2 w_2 + m_2 \ddot{w}_2 + k(w_2 - w_1) = f_2, \tag{1}$$

where $w_i = w_i(x, y, t)$ is the transverse plate displacement, $f_i = f_i(x, y, t)$ is the exciting distributed load, x, y, t are the space co-ordinates and the time, a, b, h_i are the plate dimensions, h, k are the thickness and the stiffness modulus of a Winkler elastic layer respectively, D_i is the flexural rigidity of the plate, E_i is the Young's modulus of elasticity, ν_i is the Poisson ratio, ρ_i is the mass density, $D_i = E_i h_i^3 [12(1 - \nu_i^2)]^{-1}$, $m_i = \rho_i h_i$, $i = 1, 2$,

$$\dot{w}_i = \frac{\partial w_i}{\partial t}, \quad \text{and} \quad \Delta^2 w_i = \frac{\partial^4 w_i}{\partial x^4} + 2\frac{\partial^4 w_i}{\partial x^2 \partial y^2} + \frac{\partial^4 w_i}{\partial x^4}.$$

The boundary conditions for the simply supported plates are as follows:

$$w_i(0, y, t) = w_i(a, y, t) = w_i(x, 0, t) = w_i(x, b, t) = 0,$$

$$\left.\frac{\partial^2 w_i}{\partial x^2}\right|_{(0,y,t)} = \left.\frac{\partial^2 w_i}{\partial x^2}\right|_{(a,y,t)} = \left.\frac{\partial^2 w_i}{\partial y^2}\right|_{(x,0,t)} = \left.\frac{\partial^2 w_i}{\partial y^2}\right|_{(x,b,t)} = 0, \quad i = 1, 2. \tag{2}$$

The initial conditions in general form may be written as

$$w_i(x, y, 0) = w_{i0}(x, y), \qquad \dot{w}_i|_{(x,y,0)} = v_{i0}(x, y), \quad i = 1, 2. \tag{3}$$

3. SOLUTION OF THE FREE VIBRATION PROBLEM

The free vibrations of plates are described by two homogeneous partial differential equations (1) [7, 21–23]

$$D_1 \Delta^2 w_1 + m_1 \ddot{w}_1 + k(w_1 - w_2) = 0, \qquad D_2 \Delta^2 w_2 + m_2 \ddot{w}_2 + k(w_2 - w_1) = 0. \tag{4}$$

This equation system with the governing boundary conditions (2) can be solved by the Navier method assuming the solutions in the form [7, 21–23]

$$w_1(x, y, t) = \sum_{m,n=1}^{\infty} W_{mn}(x, y) S_{1mn}(t) = \sum_{m,n=1}^{\infty} \sin(a_m x) \sin(b_n x) S_{1mn}(t),$$

$$w_2(x, y, t) = \sum_{m,n=1}^{\infty} W_{mn}(x, y) S_{2mn}(t) = \sum_{m,n=1}^{\infty} \sin(a_m x) \sin(b_n x) S_{2mn}(t), \tag{5}$$

where $S_{imn}(t), (i = 1, 2)$ are the unknown time functions:

$$W_{mn}(x, y) = X_m(x) Y_n(y) = \sin(a_m x) \sin(b_n y), \tag{6}$$

$$X_m(x) = \sin(a_m x), \qquad Y_n(y) = \sin(b_n y), \quad m, n = 1, 2, 3, \ldots,$$

$$a_m = a^{-1} m\pi, \quad b_n = b^{-1} n\pi, \qquad k_{mn}^2 = a_m^2 + b_n^2 = \pi^2[(a^{-1}m)^2 + (b^{-1}n)^2],$$

and $W_{mn}(x, y)$ are the known mode shapes of vibration for a simply supported single plate. These functions also satisfy the differential equations (4) as the corresponding boundary conditions (2).

Introducing the solutions (5) into equations (4) one obtains the expressions

$$\sum_{m,n=1}^{\infty} [\ddot{S}_{1mn} + (D_1 k_{mn}^4 + k)m_1^{-1} S_{1mn} - km_1^{-1} S_{2mn}] W_{mn} = 0,$$

$$\sum_{m,n=1}^{\infty} [\ddot{S}_{2mn} + (D_2 k_{mn}^4 + k)m_2^{-1} S_{2mn} - km_2^{-1} S_{1mn}] W_{mn} = 0.$$

This gives a set of ordinary differential equations for the unknown time functions

$$\ddot{S}_{1mn} + \omega_{11mn}^2 S_{1mn} - \omega_{10}^2 S_{2mn} = 0, \qquad \ddot{S}_{2mn} + \omega_{22mn}^2 S_{2mn} - \omega_{20}^2 S_{1mn} = 0, \tag{7}$$

where

$$\omega_{iimn}^2 = (D_i k_{mn}^4 + k)m_i^{-1}, \qquad \omega_{i0}^2 = km_i^{-1}, \qquad \omega_{120}^4 = \omega_{10}^2\omega_{20}^2 = k^2(m_1 m_2)^{-1}, \quad i = 1, 2,$$

and ω_{iimn} and ω_{120} denote the partial and coupling frequency of the system respectively. The solutions of equations (7) are as follows:

$$S_{1mn}(t) = C_{mn} e^{i\omega_{mn}t}, \qquad S_{2mn}(t) = D_{mn} e^{i\omega_{mn}t}, \qquad i = (-1)^{1/2}, \tag{8}$$

where ω_{mn} is the natural frequency of the system. Introducing them into equations (7) results in the system of algebraic equations for the unknown constants C_{mn}, D_{mn}:

$$(\omega_{11mn}^2 - \omega_{mn}^2)C_{mn} - \omega_{10}^2 D_{mn} = 0, \qquad (\omega_{22mn}^2 - \omega_{mn}^2)D_{mn} - \omega_{20}^2 C_{mn} = 0. \tag{9}$$

For this set of homogeneous equations to have a non-trivial solution, the determinant of the coefficients must vanish. This leads to the following frequency equation:

$$\omega_{mn}^4 - (\omega_{11mn}^2 + \omega_{22mn}^2)\omega_{mn}^2 + (\omega_{11mn}^2\omega_{22mn}^2 - \omega_{120}^4) = 0 \tag{10}$$

or

$$\omega_{mn}^4 - [(D_1 k_{mn}^4 + k)m_1^{-1} + (D_2 k_{mn}^4 + k)m_2^{-1}]\omega_{mn}^2$$
$$+ k_{mn}^4[D_1 D_2 k_{mn}^4 + k(D_1 + D_2)](m_1 m_2)^{-1} = 0. \tag{11}$$

The biquadratic algebraic equation (10), (11) has two positive, real roots $\omega_{1,2mn}^2$ [7]:

$$\omega_{1,2mn}^2 = 0{\cdot}5\{[\omega_{11mn}^2 + \omega_{22mn}^2) \mp [(\omega_{11mn}^2 - \omega_{22mn}^2)^2 + 4\omega_{120}^4]^{1/2}\}, \quad \omega_{1mn} < \omega_{2mn}. \tag{12}$$

This gives two infinite sequences of the natural frequencies $\omega_{1mn}, \omega_{2mn}$ in the form

$$\omega^2_{1,2mn} = 0{\cdot}5\{[(D_1 k^4_{mn} + k)m_1^{-1} + (D_2 k^4_{mn} + k)m_2^{-1}] \mp ([D_1 k^4_{mn} + k)m_1^{-1} + (D_2 k^4_{mn} + k)m_2^{-1}]^2 - 4k^4_{mn}(m_1 m_2)^{-1}[D_1 D_2 k^4_{mn} + k(D_1 + D_2)])^{1/2}\}. \quad (13)$$

Now the solutions (8) are written as follows:

$$S_{1mn}(t) = C_{1mn}\,e^{i\omega_{1mn}t} + C_{2mn}\,e^{i\omega_{1mn}t} + C_{3mn}\,e^{i\omega_{2mn}t} + C_{4mn}\,e^{i\omega_{2mn}t},$$

$$S_{2mn}(t) = D_{1mn}\,e^{i\omega_{1mn}t} + D_{2mn}\,e^{i\omega_{1mn}t} + D_{3mn}\,e^{i\omega_{2mn}t} + D_{4mn}\,e^{i\omega_{2mn}t}.$$

and then the time functions may be expressed in a more useful alternative form as

$$S_{1mn}(t) = \sum_{i=1}^{2} T_{imn}(t) = \sum_{i=1}^{2} [A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)],$$

$$S_{2mn}(t) = \sum_{i=1}^{2} a_{imn}T_{imn}(t) = \sum_{i=1}^{2} [A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)]\,a_{imn}, \quad (14)$$

where

$$T_{imn}(t) = A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t), \quad m, n = 1, 2, 3, \ldots, \quad (15)$$

$$a_{imn} = (D_1 k^4_{mn} + k - m_1\omega^2_{imn})k^{-1} = k(D_2 k^4_{mn} + k - m_2\omega^2_{imn})^{-1} = \omega_{10}^{-2}(\omega^2_{11mn} - \omega^2_{imn})$$
$$= \omega^2_{20}(\omega^2_{22mn} - \omega^2_{imn})^{-1}, \qquad k^4_{mn} = \pi^4[(a^{-1}m)^2 + (b^{-1}n)^2]^2, \quad i = 1, 2. \quad (16)$$

It may be noted that the coefficients a_{imn} (16) are as follows:

$$a_{1,2mn} = 0{\cdot}5\omega_{10}^{-2}\{(\omega^2_{11mn} - \omega^2_{22mn}) \pm [(\omega^2_{11mn} - \omega^2_{22mn})^2 + 4\omega^4_{120}]^{1/2}\},$$

$$a_{1mn} > 0, \qquad a_{2mn} < 0, \qquad a_{1mn}a_{2mn} = -\,m_1 m_2^{-1} = -\,\omega_{10}^{-2}\omega^2_{20}.$$

It is seen that the coefficient a_{1mn}, dependent on lower natural frequency ω_{1mn}, is always positive while a_{2mn}, dependent on higher frequency ω_{2mn}, is always negative.

Finally, the free transverse vibrations of an elastically connected simply supported double-plate complex system are described by the following formulae:

$$w_1(x, y, t) = \sum_{m,n=1}^{\infty} W_{mn}(x, y) \sum_{i=1}^{2} T_{imn}(t) = \sum_{m,n=1}^{\infty}\sum_{i=1}^{2} W_{1imn}(x, y)T_{imn}(t)$$
$$= \sum_{m,n=1}^{\infty} \sin(a_m x)\sin(b_n y) \sum_{i=1}^{2} [A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)], \quad (17)$$

$$w_2(x, y, t) = \sum_{m,n=1}^{\infty} W_{mn}(x, y) \sum_{i=1}^{2} T_{imn}(t) = \sum_{m,n=1}^{\infty}\sum_{i=1}^{2} W_{2imn}(x, y)T_{imn}(t)$$
$$= \sum_{m,n=1}^{\infty} \sin(a_m x)\sin(b_n y) \sum_{i=1}^{2} [A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)]\,a_{imn},$$

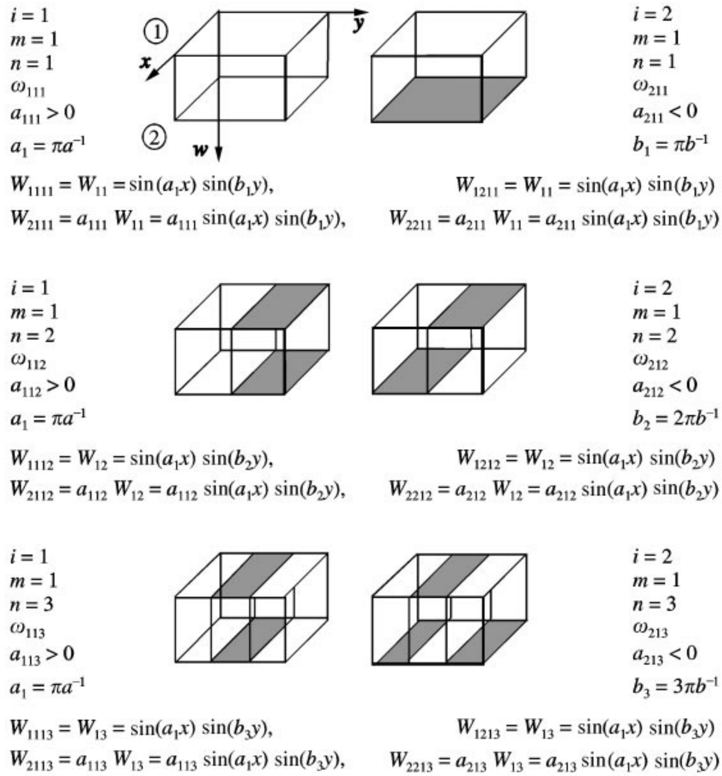

Figure 2. The general mode shapes of vibration of an elastically connected rectangular simply supported double-plate complex system corresponding to two sequences of the natural frequencies ω_{imn}, for $i = 1, 2$; $m = 1$; $n = 1, 2, 3$.

where

$$
\begin{aligned}
W_{1imn}(x, y) &= W_{mn}(x, y) = \sin(a_m x)\sin(b_n y), \\
W_{2imn}(x, y) &= a_{imn} W_{mn}(x, y) = a_{imn}\sin(a_m x)\sin(b_n y).
\end{aligned} \tag{18}
$$

The functions $W_{1imn}(x, y)$, $W_{2imn}(x, y)$ are the natural mode shapes of vibration of a plate system corresponding to two sequences of the natural frequencies $\omega_{imn}(i = 1, 2)$. The general mode shapes are presented in Figures 2–4. It is seen that an elastically connected double-plate complex system executes two types of vibrating motion: the synchronous vibrations ($i = 1$; $a_{1mn} > 0$) with lower frequencies ω_{1mn} and the asynchronous vibrations ($i = 2$; $a_{2mn} < 0$) with higher frequencies ω_{2mn}. The mode shapes obtained for simply supported plates are the same as those determined for a double-membrane system [7, 54, 57, 59, 60]. It should also be noted that the nature of the free vibrations for a simply supported double-plate system is analogous to that for a double-membrane system. The mathematical forms of the corresponding solutions are identical for both systems as a consequence of governing the same boundary conditions.

In order to determine the unknown constants A_{imn}, B_{imn} existing in expressions (17) and to find the final form of the free vibrations the initial-value problem must be solved. These constants are calculated from the assumed initial conditions (3) using the orthogonality condition of mode shape functions, which in this case has the classical

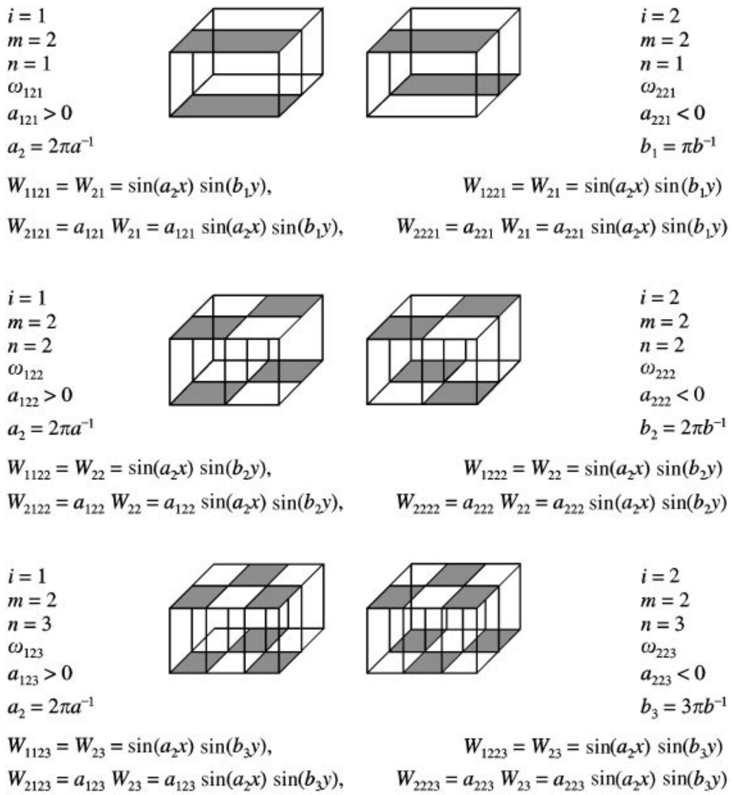

Figure 3. The general mode shapes of vibration of an elastically connected rectangular simply supported double-plate complex system corresponding to two sequences of the natural frequencies ω_{imn}, for $i = 1, 2$; $m = 2$; $n = 1, 2, 3$.

form [1]

$$\int_0^a \int_0^b W_{kl} W_{mn} \, \mathrm{d}x \, \mathrm{d}y = \int_0^a \sin(a_k x) \sin(a_m x) \, \mathrm{d}x \int_0^b \sin(b_l y) \sin(b_n y) \, \mathrm{d}y = c\delta_{klmn},$$
$$c = c_{mn}^2 = \int_0^a \int_0^b W_{mn}^2 \, \mathrm{d}x \, \mathrm{d}y = \int_0^a \sin^2(a_m x) \, \mathrm{d}x \int_0^b \sin^2(b_n y) \, \mathrm{d}y = 0{\cdot}25ab, \tag{19}$$

where δ_{klmn} is the Kronecker delta function:

$$\delta_{klmn} = 0 \quad \text{for } k \neq m \quad \text{or } l \neq n \quad \text{and} \quad \delta_{klmn} = 1 \quad \text{for } k = m \quad \text{and} \quad l = n.$$

Substituting general solutions (17) into the initial conditions (3) gives the relations

$$w_{10} = \sum_{m,n=1}^{\infty} W_{mn} \sum_{i=1}^{2} B_{imn}, \qquad v_{10} = \sum_{m,n=1}^{\infty} W_{mn} \sum_{i=1}^{2} \omega_{imn} A_{imn},$$
$$w_{20} = \sum_{m,n=1}^{\infty} W_{mn} \sum_{i=1}^{2} a_{imn} B_{imn}, \qquad v_{20} = \sum_{m,n=1}^{\infty} W_{mn} \sum_{i=1}^{2} a_{imn} \omega_{imn} A_{imn}.$$

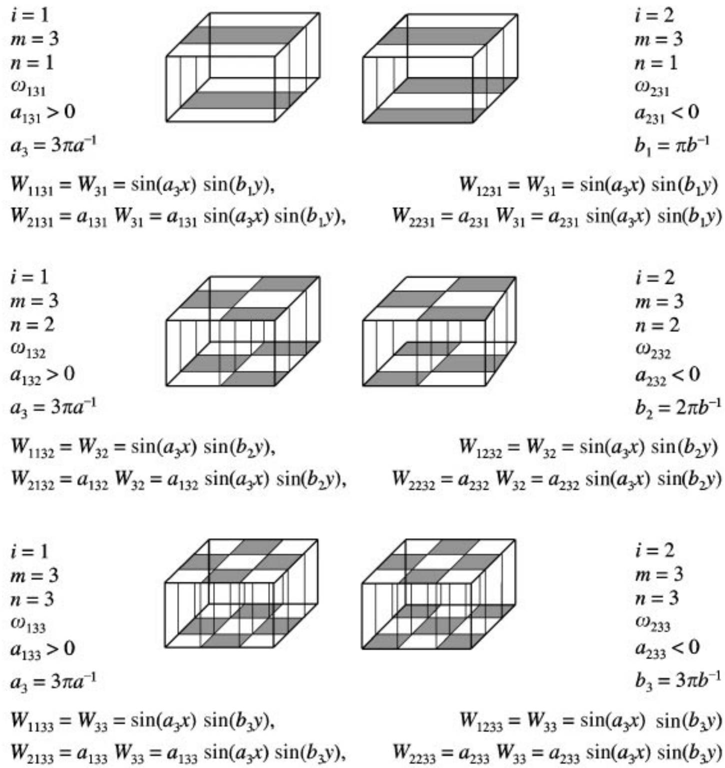

Figure 4. The general mode shapes of vibration of an elastically connected rectangular simply supported double-plate complex system corresponding to two sequences of the natural frequencies ω_{imn}, for $i = 1, 2$; $m = 3$; $n = 1, 2, 3$.

Multiplying these relations by the eigenfunction W_{kl} then integrating them over the plate surface and using the orthogonality condition (19) gives the equations, from which the following formulae, (which make it possible to calculate the unknown constants) are obtained [7, 23]:

$$A_{1mn} = (\omega_{1mn} z_{1mn})^{-1} \int_0^a \int_0^b (a_{2mn} v_{10} - v_{20}) \sin(a_m x) \sin(b_n y)\, dx\, dy,$$

$$A_{2mn} = (\omega_{2mn} z_{2mn})^{-1} \int_0^a \int_0^b (a_{1mn} v_{10} - v_{20}) \sin(a_m x) \sin(b_n y)\, dx\, dy,$$

$$B_{1mn} = z_{1mn}^{-1} \int_0^a \int_0^b (a_{2mn} w_{10} - w_{20}) \sin(a_m x) \sin(b_n y)\, dx\, dy, \tag{20}$$

$$B_{2mn} = z_{2mn}^{-1} \int_0^a \int_0^b (a_{1mn} w_{10} - w_{20}) \sin(a_m x) \sin(b_n y)\, dx\, dy,$$

where

$$z_{2mn} = -z_{1mn} = (a_{1mn} - a_{2mn})c = 0{\cdot}25ab\omega_{10}^{-2}(\omega_{2mn}^2 - \omega_{1mn}^2).$$

4. NUMERICAL EXAMPLE

The simple case of a system of two physically and geometrically identical plates is analyzed. The following values of the parameters are used in the numerical calculations:

$$a = 1\text{ m},\quad b = 2\text{m},\quad \mathrm{D} = \mathrm{D}_i = \mathrm{E}h^3[12(1-v^2)]^{-1},\quad E = E_i = 1\times 10^{10}\,\mathrm{N\,m^{-2}},$$
$$h = h_i = 1\times 10^{-2}\text{ m},\quad i = 1, 2,\quad k = 6\times 10^4\,\mathrm{N\,m^{-3}},\quad M = m_i = \rho h = 50\,\mathrm{kg\,m^{-2}},$$
$$v = v_i = 0{\cdot}3,\quad \rho = \rho_i = 5\times 10^3\,\mathrm{kg\,m^{-3}}.$$

The initial conditions are assumed to be as follows:

$$w_{10}(x, y) = w_0 \sin(a^{-1}\pi x)\sin(b^{-1}\pi y),\quad v_{10} = 0,$$
$$w_{20}(x, y) = 0{\cdot}5 w_0 \sin(a^{-1}\pi x)\sin(2b^{-1}\pi y),\quad v_{20} = 0,$$

where w_0 is an arbitrary constant.

The general solutions of the free vibrations (17) have the form

$$w_2(x, y, t) = \sum_{m,n=1}^{\infty} \sin(a_m x)\sin(b_n y) \sum_{i=1}^{2} [A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)],$$

$$w_2(x, y, t) = \sum_{m,n=1}^{\infty} \sin(a_m x)\sin(b_n y) \sum_{i=1}^{2} [A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)]\, a_{imn},$$

where the natural frequencies and the mode shape coefficients are evaluated from expressions (13) and (16) as

$$\omega_{1mn}^2 = DM^{-1}k_{mn}^4,\quad \omega_{2mn}^2 = (Dk_{mn}^4 + 2k)M^{-1} = \omega_{1mn}^2 + \omega_0^2,\quad \omega_0^2 = 2kM^{-1},$$
$$a_{1mn} = -\,a_{2mn} = 1,\qquad a_m = a^{-1}m\pi,\quad b_n = b^{-1}n\pi,\quad k_{mn}^2 = \pi^2[(a^{-1}m)^2 + (b^{-1}n)^2].$$

The results of the calculations of the natural frequencies are presented in Table 1.

The exemplary mode shapes of vibration corresponding to the first four pairs of the natural frequencies are shown in Figure 5. The natural mode shapes are described by the relations

$$W_{1imn}(x, y) = W_{mn}(x, y) = \sin(m\pi x)\sin(0{\cdot}5n\pi y),\quad i = 1, 2,$$

$$W_{2imn}(x, y) = a_{imn}W_{mn}(x, y) = a_{imn}\sin(m\pi x)\sin(0{\cdot}5n\pi y),$$

$$a_{1mn} = -\,a_{2mn} = 1,\qquad W_{mn}(x, y) = \sin(m\pi x)\sin(0{\cdot}5n\pi y).$$

Interesting and important conclusions can be drawn from the above expressions. The double-plate system executes two kinds of vibrations: the synchronous vibrations ($a_{1mn} = 1 > 0$) with lower frequencies ω_{1mn} ($\omega_{1mn} < \omega_{2mn}$) and the asynchronous vibrations ($a_{2mn} = -1 < 0$) with higher frequencies ω_{2mn}. The synchronous natural frequencies ω_{1mn} are not dependent on the stiffness modulus k unlike the asynchronous ones ω_{2mn}. The

TABLE 1

Natural frequencies of double-plate system $\omega_{imn}(s^{-1})$

m	ω_{imm}	$n = 1$ ω_{1m1} ω_{2m1}	$n = 2$ ω_{1m2} ω_{2m2}	$n = 3$ ω_{1m3} ω_{2m3}
1	ω_{11n}	52·8	84·5	137·3
	ω_{21n}	72·0	97·7	145·8
2	ω_{12n}	179·5	211·2	264·0
	ω_{22n}	186·1	216·8	268·5
3	ω_{13n}	390·7	422·4	475·2
	ω_{23n}	393·8	425·2	477·7

deflection form of each plate surface is identical for any pair of the natural frequencies ω_{imn} ($i = 1, 2$). The synchronous vibrations are performed by both plates with equal amplitudes ($a_{1mn} = 1$), and as a consequence the elastic layer is not deformed on the transverse direction. In this case, the double-plate system oscillates as a single plate with the same natural frequencies ω_{1mn}. The asynchronous vibrations are also performed with equal amplitudes ($a_{2mn} = -1$), and the natural frequencies ω_{2mn} are identical to those for a single plate vibrating on an elastic foundation of stiffness modulus $2k$.

Solving the initial-value problem the free vibrations of identical plates have the following final form:

$$w_1(x, y, t) = 0{\cdot}5 w_0 \sin(\pi x)\sin(0{\cdot}5\,\pi y)\,[\cos(\omega_{111}t) + \cos(\omega_{211}t)]$$
$$+ 0{\cdot}25 w_0 \sin(\pi x)\sin(\pi y)[\cos(\omega_{112}t) - \cos(\omega_{212}t)],$$
$$w_2(x, y, t) = 0{\cdot}5 w_0 \sin(\pi x)\sin(0{\cdot}5\,\pi y)\cos(\omega_{111}t) - \cos(\omega_{211}t)]$$
$$+ 0{\cdot}25 w_0 \sin(\pi x)\sin(\pi y)[\cos(\omega_{112}t) + \cos(\omega_{212}t)].$$

The assumed initial conditions generate the vibrations of the system with the first two pairs of the natural frequencies, i.e., $\omega_{111}, \omega_{211}$ and $\omega_{112}, \omega_{212}$. The plates execute the synchronous vibrations with lower frequencies $\omega_{111} = 52{\cdot}8(s^{-1})$ and $\omega_{112} = 84{\cdot}5(s^{-1})$, and the asynchronous vibrations with higher frequencies $\omega_{211} = 72{\cdot}0(s^{-1})$ and $\omega_{212} = 97{\cdot}7(s^{-1})$ (see Figure 5).

5. CONCLUSIONS

The free transverse vibration theory of an elastically connected simply supported double-plate complex system is developed. The solutions of the differential equations of motion are formulated by the classical Navier method. The natural frequencies of the system in the form of two infinite sequences $\omega_{1mn}, \omega_{2mn}$ ($\omega_{1mn} < \omega_{2mn}$) are determined and the corresponding mode shapes of vibrations are shown. The free vibrations of a double-plate are realized by two kinds of motions: the synchronous vibrations ($a_{1mn} > 0$) with lower frequencies ω_{1mn} and the asynchronous vibrations ($a_{2mn} < 0$) with higher frequencies ω_{2mn}. The initial-value problem is considered to find the final form of the free vibrations. It can be noted that the nature of the free vibrations for a simply supported double-plate system and for a double-membrane system [7, 54, 57, 59, 60] is similar. It can

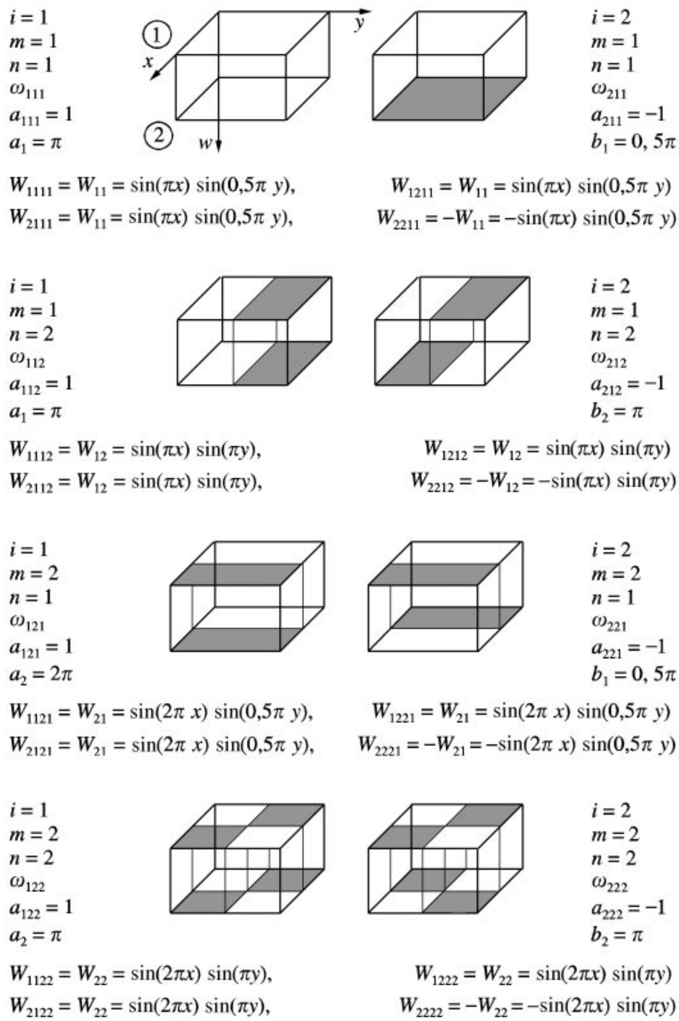

Figure 5. The mode shapes of vibration of a system of two elastically connected rectangular simply supported identical plates corresponding to the first four pairs of the natural frequencies.

be also shown that the corresponding two-degree-of-freedom complex discrete system described in reference [63] is an analogue of an elastically connected double-body complex continuous system represented, for example, by a double-string system [62, 63], double-beam system [64], double-membrane system [60], and the double-plate system presented here. A plate supported on an elastic foundation is a particular case of the double-plate system considered. The solution procedure applied in this paper can be used in the investigation of a general elastically connected multi-plate complex system [7, 20, 49, 50].

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